Disc. 108-S09/From the January-February 2011 *ACI Structural Journal*, p. 80

Empirical Equations for Peak Shear Strength of Low Aspect Ratio Reinforced Concrete Walls
by C. Kerem Gulec and Andrew S. Whittaker

Discussion by Gilbert H. Béguin

ACI member, PhD, Grandson, Switzerland

The derivation of an empirical equation for the peak shear strength of deep beams is a worthy endeavor; however, the discusser wishes to call attention to the following:

- In such a deep girder, the distribution of shear stresses is different from that in a conventional beam (Navier's theory);
- As shown by Chow et al.,[14] Bay,[15] and Dischinger,[16] in vertical sections, there is a stress concentration in the neighborhood of the supports; and
- The width *d* of the support is a critical parameter.

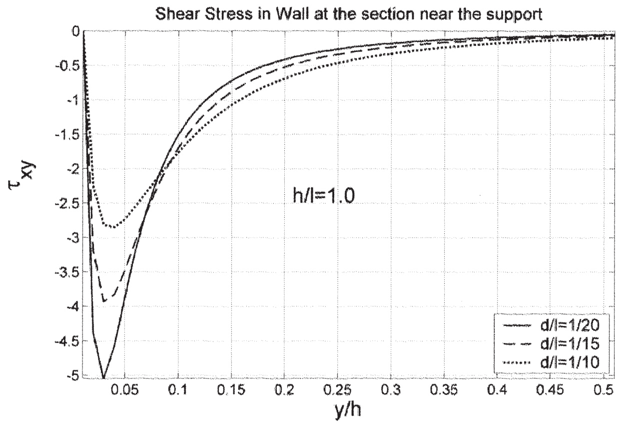

Fig. 18—Shear stress distribution in deep beam with h/l *= 1.0 in vertical section tangent to supporting element.*

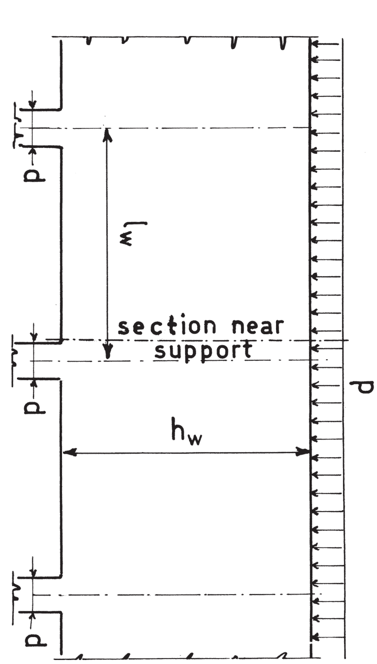

Fig. 19—Continuous beam on many supports (notation and position of section considered).

To illustrate this dependence, the discusser has computed—within the two-dimensional theory of elasticity—the shear stresses in a vertical section near the support of a continuous deep beam on several supports under a uniform load. Various cases have been examined: 1) $h_w/l_w = 1.0$, with $d/l_w = 1/20$, 1/15, and 1/10, respectively; and 2) $h_w/l_w = 1.5$, with $d/l_w =$ 1/20, 1/15, and 1/10, respectively.

Figures 18 and 20 show the results of the computation. The thickness of the deep beam and the applied uniform load *p* have been taken as 1. It follows that for a real case, the values on the graphs must be multiplied by the applied pressure *p* in kN/m (kip/ft) and divided by the thickness in m (ft) to obtain a stress expressed in kN/m^2 (kip/ft^2). The maximum values of the shear stress as computed are as follows: 1) $h_w/l_w = 1.0$ and $\tau_{max} = -5.14$, -3.92, and -2.85, respectively; and 2) $h_w/l_w =$ 1.5 and $\tau_{max} = -4.95$, -3.83, and -2.85, respectively.

Parkus[17] has studied deep beams on three supports. He concludes that "the stress distribution in this case is essentially like that found in a deep girder on infinitely many supports" (Fig. 19).

The discusser thinks that the large scatter of these data may be partially due to the differing widths of the supports in the reported tests.

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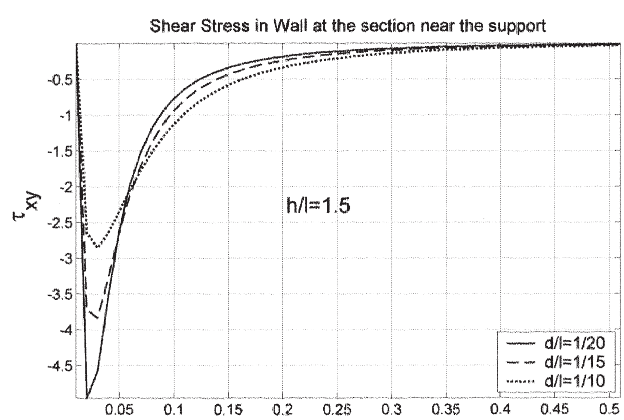

Fig. 20—Shear stress distribution in deep beam with h/l *= 1.5 in vertical section tangent to supporting element.*

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Empirical Equations for Peak Shear Strength of Low Aspect Ratio Reinforced Concrete Walls

by C. Kerem Gulec and Andrew S. Whittaker

Discussion by Himat Solanki and Sonal Thakkar

ACI member, Sarasota, FL, and Assistant Professor, Nirma University, Ahmedabad, India

The authors have presented an interesting paper on empirical equations for the peak shear strength of low aspect ratio reinforced concrete (RC) walls; however, the discussers would like to offer the following comments:

1. The authors mentioned that the aspect ratio was the most influential parameter, but it appears that they have not considered the ACI 318-08[3] recommendation for the rectangular walls. This means that the area of the wall was considered the entire width of the wall, where there were no boundary elements—that is, columns or flanges.

2. Equations (4) and (5) do not include the contribution of horizontal reinforcements. In fact, the horizontal reinforcement increases the shear strength up to 20%, depending on the amount of horizontal reinforcement. The authors' concept appears to be inconsistent with the strut-and-tie model (STM). Please refer to Fig. 21.

3. The discussers have studied numerous squat walls with a horizontal reinforcement ratio ρ_h of 0.23 to 1.26%, $P/A_w f_c'$ of 0 to 0.271, and an effective width of $0.8l_w$ of the rectangular walls (ACI 318-08[3] recommendation), as well as dynamic tests on RC shear walls. Using the aforementioned assumptions, the discussers have analyzed walls as outlined in References 5, 6, 18, and 19. Based on the analysis, the mean value and the coefficient of variation were found to be 1.08 and 0.247, respectively.

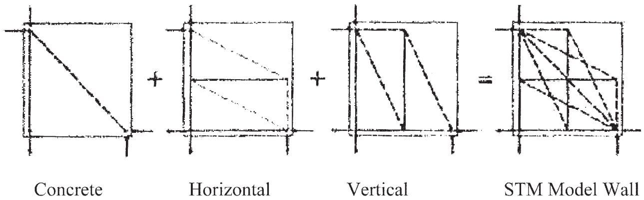

Fig. 21—STM of squat walls.

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AUTHORS' CLOSURE

We thank the discussers and appreciate the opportunity to respond.

Béguin comments

The authors agree with the comment regarding the disturbed stress field near the support but note that elasticity-based solutions are not relevant for predicting the peak shear strength of RC walls.

The boundary conditions for low aspect ratio RC walls are different from those associated with two-span continuous deep beams, as described by the discusser. Most of the walls in the database were tested as cantilevers with an upper loading beam that was free to rotate and a stiff foundation fixed to a strong floor.

Solanki and Thakkar comments

Three comments were made. The authors' responses are listed as follows:

1. The predictive equations of Chapters 11 and 21 of ACI 318 ignore the effect of boundary elements on the shear strength of RC walls. The authors investigated the performance of different predictive equations for the peak shear strength of low aspect ratio rectangular walls and walls with boundary elements.[7,8] A database of 400+ tests was assembled for the assessment of the predictive equations. We observed significant scatter in the ACI 318 predictions of peak shear strength in the 400+ walls and note that the ACI 318 equations provided a ratio of predicted-to-measured peak shear strength of approximately 1.0 in the average sense for rectangular walls[7] but considerably underestimated the peak shear strength of walls with boundary elements.[8]

2. On the basis of nonlinear regression on the test data, the authors concluded that the effect of horizontal web reinforcement on the peak shear strength of low aspect ratio walls is relatively insignificant, especially when compared with the contributions from other design parameters. There is experimental evidence to support this observation,[18,20-22] but the data are not conclusive. The authors investigated this observation further by performing a series of parametric studies using finite element analysis.[2] These studies showed the effect of the horizontal web reinforcement ratio on the peak shear strength of low aspect ratio walls to be small. These studies were not described in the paper due to space limitations.

3. The authors' statistics of the ratio of predicted-to-measured peak shear strength were presented in Tables 6 and 7 of the paper for rectangular walls and walls with boundary elements, respectively. For these calculations, databases of 74 rectangular walls and 153 walls with boundary elements were used. Detailed information on the 400+ wall database is provided in Reference 2.

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Disc. 108-S11/From the January-February 2011 *ACI Structural Journal*, p. 99

Failure Mode and Ultimate Strength of Precast Concrete Barrier

by Se-Jin Jeon, Myoung-Sung Choi, and Young-Jin Kim

Discussion by Himat Solanki and Anand Mehta

ACI member, Sarasota, FL, and Gandhinagar, Gujarat, India

The authors have presented an interesting paper on the failure mode and ultimate strength of a precast concrete barrier; however, the discussers would like to offer the following comments:

1. Based on the test, the authors considered the loading height at the center of gravity (CG) of the vehicle in lieu of the bumper height, which normally plays a major role during the impact. The height and width of the loading (810 or 1070 mm [31.89 or 42.13 in.] and 1070 mm [42.13 in.], respectively) are inconsistent with Table 13.7.2-1 in References 16 and 24.

2. Normally, trucks and/or buses have a low-frequency of average daily traffic (ADT) volume as compared to passenger vehicles. Therefore, it is very important to simulate the passenger vehicle-barrier interaction.

3. The authors' static tests were based on the frontal impact test; however, Reference 16 suggests using an oblique angle from 15 to 25 degrees. An oblique angle will result in a different yield line pattern as compared to frontal tests.

4. The ultimate strength due to static load tests does not consider the initial stiffness, the stiffness after the impact of the vehicle, and the impact loading duration time. Based on the National Highway Transportation Safety Administration's (NHTSA's) test study, the ratio of initial stiffness to the stiffness after the impact would be in the range of 10 or greater and the impact duration time would be in the range of 0.085 to 0.100 seconds.

5. The ultimate test values presented in Table 2 are unclear. Did these values consider the effect of anchorages, as shown in Fig. 3? The effect of anchorages in precast concrete barriers is very important and cannot be ignored.[25,26]

6. The authors' ultimate strength values are based on the static test; however, these values do not represent the dynamic effect that normally happens due to impact. Because the dynamic magnification factor is greater than 1.0 (in a range of 1.4 to 1.6), the values in Table 2 require some modification.

7. The authors' yield line pattern for the static frontal test is not a new development. It can be classified as a cantilever slab with a partial knife-edge line loading condition. This condition can be found in many textbooks and published research papers on the yield line theory.

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AUTHORS' CLOSURE

The authors would like to thank the discussers for their interest in the paper and have provided clarifications to the comments raised as follows:

1. Before responding to the comments, the authors would like the readers to know that the fifth edition of the *AASHTO LRFD Bridge Design Specifications* has been published recently. Although the fourth edition[16] was referenced in the paper, the content related to railings or barriers is the same in these two editions. First, it should be clarified that the discussers mentioned Table 13.7.2-1, which is used for a vehicle crash test, whereas this study referred to the equivalent design forces of traffic railings presented in Table A13.2-1 to perform a quasi-static test. No mistake is made in the paper in terms of incorporating Table A13.2-1 into the test.

By comparing Table A13.2-1 with Table 13.7.2-1, it can be identified that the main design forces are applied not at the CG of a vehicle but at somewhere below the CG. The authors believe that the bumper height of the vehicle is incorporated into H_e of Table A13.2-1. On the other hand, the forces applied at the height of the vehicle's CG are used to determine the effective height of a railing to prevent vehicle rollover, as shown in Fig. CA13.2-1.

The following is a repetition of the loading pattern section of the paper but is presented for clarification. Because the load was applied at the top of the barrier, the loading height was approximately 1320 mm (51.97 in.), which satisfies the minimum loading heights of Table A13.2-1 for both test levels (810 and 1070 mm [31.89 and 42.13 in.] for Test Levels TL-4 and TL-5, respectively) considered in this study. On the other hand, the lengths of the loading were 1070 and 2440 mm (42.13 and 96.06 in.) for Test Levels TL-4 and TL-5, respectively, according to Table A13.2-1.

The authors are also aware of the prEN 1317 "Road Restraint Systems"[24] the discussers mentioned, where a variety of performance classes are presented, similar to the test levels of the AASHTO LRFD specifications.[16] It should be noted, however, that the equivalent design forces as presented in Table A13.2-1 of the AASHTO LRFD specifications[16] are not provided in prEN 1317. This is why prEN 1317 is not referenced in the paper.

2. The test levels of railings are selected based on the types and proportions of the vehicles anticipated on the road concerned, as stated in Section 13.7.2 of the AASHTO LRFD specifications[16] regarding each test level. Although passenger vehicles occupy a major portion of the traffic on most roads, the design forces for railings, which represent the required strength of the railings, are mainly derived from heavier vehicles, such as trucks, buses, and tractor-trailers. For instance, it can be seen in Table 13.7.2-1[16] that a single-unit van truck and van-type tractor-trailer are taken into account for Test Levels TL-4 and TL-5, respectively, resulting in the design forces of Table A13.2-1.[16]

The authors agree that the vehicle-barrier interaction is an important factor affecting structural adequacy, occupant risk, and vehicle trajectory. The interaction is accounted for in the vehicle crash test and computer simulation, and it is incorporated in an approximate way in the design forces of

Table A13.2-1, as stated in Section CA13.2 of the AASHTO LRFD specifications.[16]

3. The design forces of Table A13.2-1[16] may not be derived by assuming a normal impact, and the crash angles of approximately 15 to 25 degrees presented in Table 13.7.2-1[16] are accounted for. A similar procedure can be found in a formula proposed by Olson et al. and presented in NCHRP *Report* 86,[27] which is used to convert the vehicle crash effect into the equivalent transverse force applied to a railing. It can be seen that the crash angle is included in the formula.

The transverse forces of Table A13.2-1 were considered in the test because they are the dominant factors affecting the yield line pattern and ultimate strength of the barrier, as explicitly demonstrated in Section A13.3.1 of the AASHTO LRFD specifications.[16] It should be noted that the longitudinal forces of Table A13.2-1 are not directly related to the crash angles and are derived from the transverse forces and the friction coefficient between the barrier and a vehicle. It can be seen that the friction coefficient is assumed to be 0.333 in Table A13.2-1. In this respect, it should be noted that the oblique angles do not directly affect the yield line pattern of a barrier.

4. The authors do not insist that the static test of the barrier can represent all the aspects and structural behavior anticipated in the real crash of a vehicle. The purpose and usefulness of the static test compared to a final verification through the vehicle crash test were addressed in the introduction of the paper. The ratio of the two stiffnesses of a vehicle before and after the impact and the contact time during the impact are related to the vehicle crash test and computer simulation.

5. As shown in Fig. 2, the loop splice and mortar filling were the main tools of this study to ensure a robust joint between the precast concrete barriers and deck. As was addressed in the paper, the joint maintained a reasonable integrity up to the ultimate loads presented in Table 2. Without these types of anchorage systems, the precast barriers would turn over or move outward when subjected to a vehicle crash and not attain a required ultimate load.

6. As has been repeatedly mentioned in response to the previous comments, the test of this study follows the procedure presented in Table A13.2-1 of the AASHTO LRFD specifications.[16] Although the dynamic magnification factor is outside of the scope of this study, it should be noted that the impact velocity of a vehicle is taken into account in deriving the transverse design forces of Table A13.2-1, as is the case in the aforementioned formula of NCHRP *Report* 86.[27] This implies that at least some of the dynamic aspects of a vehicle crash are accounted for in establishing the equivalent design forces corresponding to the test levels. The authors believe, therefore, that reducing the magnitudes of the ultimate loads of Table 2 in consideration of the dynamic magnification factor is inconsistent with the usual procedure to determine the test level of the barrier according to Table A13.2-1.

7. A number of yield line patterns have been proposed for a variety of structural shapes, boundary conditions, loading patterns, and so on. The yield line pattern of this study has been proposed for the barriers that are longitudinally continuous and have a tapered section with some points of slope discontinuity (as used worldwide), whereas most of the yield lines that can be found in the previous studies or textbooks deal with a structure with a constant thickness. The authors believe that the experimental and analytical attempts to improve the conventional yield line shape presented in the AASHTO LRFD specifications[16] should be considered, rather than the shape of the yield line itself.

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Disc. 108-S12/From the January-February 2011 *ACI Structural Journal*, p. 108

Distribution of Stirrups across Web of Deep Beams

by Robin Tuchscherer, David Birrcher, Matthew Huizinga, and Oguzhan Bayrak

Discussion by Rafael Alves de Souza, João da Costa Pantoja, and Luiz Eloy Vaz

ACI member, Associate Professor, Universidade Estadual de Maringá, Maringá, Brazil; Assistant Professor, Universidade Federal Fluminense, Rio de Janeiro, Brazil; Associate Professor, Universidade Federal Fluminense

The authors have presented the results of deep beams subjected to shear to evaluate the benefit of distributing stirrups across the web. By testing three full-scale deep beams using a very interesting procedure and adopting the number of stirrup legs distributed across the web and the amount of web reinforcement as the primary experimental variables, the authors were able to obtain a total of six tests. Based on these tests, the authors have concluded that the addition of closely spaced stirrups did not significantly improve the shear capacity or serviceability performance of deep beams with a shear span-depth ratio (a/d) of 1.84 or 1.85. Despite the quality of their research, some additional issues should be discussed to clarify some topics and enhance the entire comprehension of this interesting paper.

INTRODUCTION

The authors state that the assumptions of a linear-elastic analysis usually assumed for designing beams are not valid for deep beams; therefore, another analytical method, such as the strut-and-tie model (STM), must be employed. In fact, there are other available methods that could be used for this task, such as stress fields,[13] the stringer panel model,[14] and finite element procedures optimized for membrane action design[15] and analysis.[16] Strut-and-tie modeling can undoubtedly provide fast solutions for engineers when compared to the other alternatives.

Unfortunately, the authors do not provide clear information regarding the deep beam behavior and, perhaps for this reason, some difficulties arise when interpreting their results based on the STM approach.

A deep beam is a beam with a large depth-thickness ratio and a short a/d ($a/d < 2.0$); therefore, its behavior is completely different from that expected for slender or intermediated beams. Deep beams present two-dimensional behavior, whereas ordinary beams present one-dimensional behavior (B-region, beam, or Bernoulli region). Also, the assumption of plane sections is not valid, as the shear deformation cannot be neglected (D-region or disturbed region) in deep beams.

As stated by the authors, the mechanism of shear transfer predominantly results from compressive stresses flowing directly from the load to the support; therefore, the capacity of a simple deep beam is dependent on the compressive strength of the concrete in the strut. As the shear transference is mainly made by a concrete strut and a tension tie, the authors are right in their conclusions regarding vertical stirrups across the web of deep beams—that is, for deep beams, the transverse reinforcement is only necessary for cracking control and for improving deformation capacity. In the discussers' opinion, horizontal stirrups distributed across the web could work better than vertical stirrups, as these can be more effective for controlling tensile strains in the bottle struts. What do the authors think about this opinion, taking into account their experimental experience?

The discussion about one- or two-panel behavior for shear transference is good, but it should include more significant details. The authors did not explain, for example, the "arch effect" that frequently occurs to better explain the shear transference in deep or slender beams with concentrated loads near the supports. Because of the "arch effect," the tensile force in the longitudinal reinforcement of deep beams is constantly maintained, as seen in pile caps, dapped beams, and corbels. This behavior is completely different for a slender beam, where the tensile force in the longitudinal reinforcement presents variations along the beam, whereas the internal level arm is kept constant. This information would be useful, for example, to better explain the experimental results section and the conclusion that the distribution of vertical reinforcement across the web of a deep beam has a minor influence on the shear capacity.

The authors state that when the a/d exceeds a value of 2, the mechanism of shear failure is better characterized by a sectional shear (beam model), as the shear resistance of the beam is dependent on the cross section and the tensile resistance of the vertical stirrups. The authors are right, but in fact it is just a simple suggestion of limit value based on the Saint-Venant's principle, as it is difficult to propose a generalization of transition (deep beam behavior to slender beam behavior). Despite this problem, could the authors indicate for the assumed transition situation ($a/d = 2$) which model would demand more longitudinal reinforcement? In the discussers' opinion, for that situation, the effective depth may overestimate the shear strength and could demand more flexural reinforcement while using a beam approach.

RESEARCH SIGNIFICANCE

As mentioned by the authors, there is not a consensus as to whether the spacing of stirrups should be limited across the web of a deep-beam region. They also mentioned that past research has examined this matter for beams with an a/d greater than 2, but similar studies have not been conducted for deep beams. In the discussers' opinion, there is no research on this topic because the vertical stirrups have only had a minor importance in the shear strength of deep beams[17] since the 1960s. Also, ensuring equilibrium and assuming that the longitudinal reinforcement will experience yielding before the crushing of the diagonal concrete struts is a simple condition for obtaining a collapse load higher than the design load, as provided by the lower-bound theorem of the theory of plasticity.[18-20] Therefore, in the discussers' opinion, the web reinforcement applied to D-regions is needed just to better control the cracking propagation or enhance critical bottle struts with additional horizontal stirrups.

EXPERIMENTAL PROGRAM AND EXPERIMENTAL RESULTS

The test setup section explains that the beams were monotonically loaded in approximately 50 kips (220 kN) and that for each load increment, the maximum width of any diagonal crack was recorded on both sides of the shear span under investigation. In the discussers' opinion, it is a very large step in a way that would be difficult to understand the crack width evolution without an abrupt variation. Could the authors explain how they determined this large load step?

Regarding the strength results section, the authors state that the failure of each test region was typically preceded by the crushing of concrete in the nodal region adjacent to the load plate and, therefore, it was more appropriate to normalize the shear capacity by the compressive strength of the concrete than the square root of the compressive strength. Could the authors better explain this last assertion and how to analyze the meaning of the last columns in Table 3?

SUMMARY AND CONCLUSIONS

The authors have presented a very interesting paper concerning the behavior of deep beams and they should be complimented on their research. Based on the test results, the authors were able to demonstrate that web reinforcement and the number of stirrups slightly influence the shear strength of deep beams with an a/d of less than 2. In fact, the obtained results could already be expected, taking into account the use of an STM. Taking into account the lack of experimental results for the deep beams with an a/d of less than 2, however, the authors had an opportunity to extend the data bank that is available for deep beams. The authors are encouraged to research the application of steel fibers and passive spiral confinement reinforcement for the diagonal struts, as the authors are concerned about the enhancement of the shear strength of deep beams.

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AUTHORS' CLOSURE

The authors thank the discussers for their comments and offer a few comments to close the discussion.

1. The discussers are reminded that, within limited space available for a technical paper, as the authors, we stated the objectives for the paper, presented experimental facts that guided our thinking, and reached conclusions that were based on experimental evidence and consistent with the objectives. As such, the authors will only comment on the facts presented in the subject paper. The authors will not speculate on some experimental variables that have not been studied.

2. The discussers suggest that horizontal stirrups may be more effective than vertical stirrups at controlling tensile strains in bottle-shaped struts. This topic was purposely not discussed in the subject paper because it is relatively complex and deserves more attention than permitted by the length requirements. With that said, the discussers are referred to the commentary of ACI 318-08,[5] Sections R11.7.4 and R11.7.5, which state that "tests have shown that vertical shear reinforcement is more effective than horizontal shear reinforcement." The authors conducted a database analysis of previous deep beam shear tests and were able to further substantiate the aforementioned commentary.[4] The authors refer the discussers to the authors' research report[4] for further information regarding the effectiveness of reinforcement in deep beams.

3. The specimens presented in this paper were specifically configured to evaluate the effectiveness of distributing transverse reinforcement across a beam's web. Again, the authors refer the discussers to the authors' research report[4] for information with respect to the effectiveness of the reinforcement ratio and the transition region between deep and slender beam behavior. These topics are complex and nuanced and deserve much more attention than could be discussed within the limitations of the subject paper.

4. As spelled out in the paper, the authors are in agreement with the discussers regarding the effectiveness of shear reinforcement in deep beams with respect to strength. The effectiveness of distributing shear reinforcement across the web of a deep beam, however, is an important topic in view of the fact that AASHTO LRFD specifications[7] (Fig. 4) require a minimum amount of distribution, and there is a sparse amount of guidance provided elsewhere. The implications of stirrup detailing on serviceability are a different issue that was discussed in the paper.

5. The authors selected a load step equal to 10% of the expected final load, thereby resulting in at least 10 load increments until failure. Given the variability inherent in crack measurements, the load increments were deemed sufficient for determining the overall trends in crack width propagation.

6. Experimental loads that are associated with the tensile strength of concrete, such as the diagonal cracking load or the sectional shear (that is, diagonal tension) strength of a member, are typically normalized by $\sqrt{f_c'}$. Experimental loads that are associated with the compressive strength of concrete, such as the ultimate capacity of a deep beam, are typically normalized by f_c'. The authors based the findings of this study on the experimental results normalized by f_c'. Recognizing that many practitioners are familiar with shear values normalized by $\sqrt{f_c'}$, however, both types of values were presented with the results.